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COMPOSITE LAMINATE WEIGHT OPTIMIZATION ON THE HX-20

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An automated method is presented for sizing composite laminate to get minimum total thickness. The method can handle multiple, independent inplane loads. Sizing is based on strength requirements. A listing is presented in BASIC for an Epson HX-20 microcomputer.

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FOREWORD

This report was prepared in the Mechanics and Surface Interactions Branch (AFWAL/MLBM), Nonmetallic Materials Division, Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio. The work was performed under the support of Project Number 2307, "Nonmetallic Structural Materials", Task Number 2307P2, "Composite Materials and Mechanics Technology".

In this report, an automated composite laminate sizing technique is presented, which optimizes for minimum weight. The technique can be coded for a microcomputer, and a listing in BASIC is given for an Epson HX-20 microcomputer. The program is interactive and easy to use. The optimization is for point stress under multiple loads.

The program is available on an audio cassette, and can be obtained by sending a blank tape to AFWAL/MLBM, Wright-Patterson AFB, Ohio 45433 and referencing this report.

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SECTION I

PROGRAM DESCRIPTION

CLASS (Composite Laminate Automated Sizing for Strength) is an interactive optimization program designed to run on a small microcomputer. The listing presented here is for an Epson HX-20 portable computer with 16K of memory. The version of Basic is standard enough that translations to other microcomputers would be easy.

The program will find a minimum thickness laminate which will not fail under any of the load conditions entered. Ply orientations are chosen by the user. The program's capability in handling multiple, independent loads could be useful for loads which change with time or position on a constant thickness plate, or for situations where there is uncertainty in calculating the loads. As the program is currently dimensioned, four independent load combinations and 6-ply orientations can be entered.

Only point stresses are considered, thus the program optimizes the laminate only at one point in the structure. Furthermore, the program assuems in-plane loads only and no out-of-plane deflections. This implies a symmetric laminate, but stacking sequence is not a factor in the program. The layer thicknesses generated by the program are the total and must be divided by 2 to get the halves of a symmetric laminate.

No knowledge of optimization techniques is needed to run the program and very little knowledge of laminate plate theory is needed. In addition, material properties for five common advanced composites are stored in the program, or the computer can ask for new properties through prompts.

SECTION II

GENERAL INSTRUCTIONS

The program can be entered by hand from the listing given, or by an audio cassette tape available from AFWAL/MLBM. If entered by hand, material properties can be loaded by running the program and using the "NEW" material option. To load the tape, simply hook-up to cassette as shown in the Epson instruction manual, and enter LOAD "CLASS", R. The "R" is needed to automatically start a part of the program which in turn loads the material properties from the tape. Material properties are stored in the Epson's "Ram File" feature which allows the machine to be turned off without losing data.

Running the program on another computer should be possible if 13K of memory is available. The commands most likely to need changing are PUT% and GET% used for the Epson's Ram File operations. Equivalent disk commands or data statements can be used instead.

When running, the program prompts the user for input information. An example dialogue between computer and user is given below along with the printed output generated.

SECTION III

COMPUTER/USER DIALOGUE

LCD Display	Keyboard Response (comments in parenthesis)
Press any key when desired Material appears	RUN RETURN (unless otherwise noted, "Return" key pressed after each keyboard entry)
T300/5208	
B(4)/5505	
AS/3501	
Scotchply 1002	
Kevlar 49/Epoxy	
Aluminum	
New	(random key pressed when "New" appears on screen)
REVIEW OR NEW DATA (R/N) ?	N
WHICH MATERIAL WILL YOU REPLACE (0-5) ?	5 (materials numbered in same order as listed T300/5208=0)
EX(GPa) = ?	185
EY(GPa) = ?	6.76
VX = ?	.2
ES(GPa) = ?	5.86
X(MPa) = ?	680
X'(MPa) = ?	690 (primed constants imply compressive properties)
Y(MPa) = ?	16
Y'(MPa) = ?	186
S(MPa) = ?	72
THICKNESS (m.) = ?	125E-6 (ply thickness)
NAME (15 CHR MAX) ?	HMS/3002M
ADDITIONAL CHANGES ?	N

LCD Display	Keyboard Response (comments in parenthesis)
. . . .	(Materials list begins again, this time with the new material replacing aluminum, when it appears a key is pressed)
HOW MANY PLY GROUPS ?	4
ENTER PLY GROUP	
ORIENTATIONS	
PLY 1 = ?	0
PLY 2 = ?	90
PLY 3 = ?	45
PLY 4 = ?	-45
ENTER NUMBER OF INDEPENDENT LOADING CONDITIONS ?	2
LOAD 1 in MPa	
N1 = ?	3
N2 = ?	2
N6 = ?	.5
LOAD 2 in MPa	
N1 = ?	1
N2 = ?	4
N6 = ?	0
WORKING ITERATION 1	(after 4 iterations and about 7 minutes the computer beeps that the solution has been found. This example ran for an unusually long time. Most problems will run in less time)
TOTAL THICKNESS = 1.71342 E-02 m. 137.07 PLIES HIT ANY KEY TO CONTINUE	
Press Y if printout of displayed result is desired. Press N if not	(press any key, no return)

Material Properties
HMS/3002M
EX= 185 GPa
EY= 6.76 GPa
ES= 5.86 GPa
VX= .2
X= 680 MPa
X'= 690 MPa
Y= 16 MPa
Y'= 186 MPa
S= 72 MPa
Ply Thickness .000125 m

LOADING 1
N 1= 3 MN/m
N 2= 2 MN/m
N 6= .5 MN/m
LOADING 2
N 1= 1 MN/m
N 2= 4 MN/m
N 6= 0 MN/m

Total thickness=
.0171E+00 m.
137.07 Plies

ANGLE	RATIO	#PLIES
0	.3476	47.65
90	.5281	72.38
45	.1243	17.04
-45	0	0

STRENGTH RATIOS
1=ULTIMATE STRAIN
>1 IS SAFE
LOADING 1

PL'	
0	1.1622
90	1
45	1.1616

LOADING 2

PLY	
0	1.0054
90	1.2115
45	1.0122

LAMINATE STRAINS
LOADING 1
e1=+2.182E-03
e2=+0.902E-03
e6=+1.096E-03
LOADING 2
e1=+0.697E-03
e2=+2.216E-03
e6=-1.467E-03

Norm. |A| in GPa.

74.759	6.511	5.549
6.511	106.969	5.549
5.549	5.549	11.017

Compliance (normalized) in 1/TPa.

13.922	-0.497	-6.762
-0.497	9.617	-4.594
-6.762	-4.594	96.490

Ply Ratio Graph

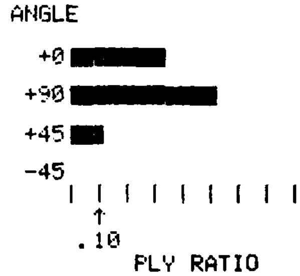


Figure 1. Output Produced from Example Input Dialogue

LCS Display	Keyboard Response (comments in parenthesis)
PLY PROPERTIES	Y (return key not used for these responses)
LOADS	Y
TOTAL THICKNESS & PLY RATIOS	Y
STRENGTH	Y
LAMINATE STRAINS	Y
STIFFNESS MATRIX	Y
COMPLIANCE MATRIX	Y
PLY RATIO GRAPH	Y (after entire list of print-out options is presented, computer produces the print-out shown on next page)
FINISHED HIT ANY KEY TO CONTINUE	(pressing a key restarts program. Press "BREAK" key to exit).

SECTION IV

METHOD

The goal is to minimize the total thickness of a composite laminate subject to failure constraints under static loads. Specifically,

$$\sum_{i=1}^{m} h_i = \min \quad \text{where } m = \text{number of ply groups}$$

subject to $h_i \geq 0$

and $G_{jk}^{(\theta_i)} \varepsilon_j^{(L)} \varepsilon_k^{(L)} + G_j^{(\theta_i)} \varepsilon_j^{(L)} - 1 \leq 0$ where h_i is the total thickness of all the plies at the kth orientation (which will be referred to as a "layer" in this report). The failure criteria is a first ply failure based on the Tsai-Wu tensor criteria in strain space. The G's are transformed to the laminate axis from the ith layer's orientation. The strains are associated with the Lth loading combination. This distinction is made since more than one independent loading may be considered. For the definition of the G's in terms of experimental strength data, see Reference 1.

Stacking sequence is not included in this formulation, and the laminate is assumed not to bend or warp. Therefore, strains and loads are related by

$$\vec{N} = |A| \vec{\epsilon}$$

The optimization method applied is a modification of the method of feasible directions (Reference 2). The method can be demonstrated graphically with two-dimensions, i.e. two layers. In Figure 2 the two equalities

$$G_{ij}^{(0)} \epsilon_i \epsilon_j + G_i^{(0)} \epsilon_i - 1 = 0$$

$$G_{ij}^{(90)} \epsilon_i \epsilon_j + G_i^{(90)} \epsilon_i - 1 = 0$$

have been ploted as functions of $h^{[0]}$ and $h^{[90]}$ for the single loading condition shown. Any point above and to the right of these two curves is feasible, that is, failure will not occur. Points to the left and below the curves are infeasible. Because our objective function (the sum of the layer thicknesses) is linear, the optimum point will lie on one of these curves or the intersection of multiple curves.

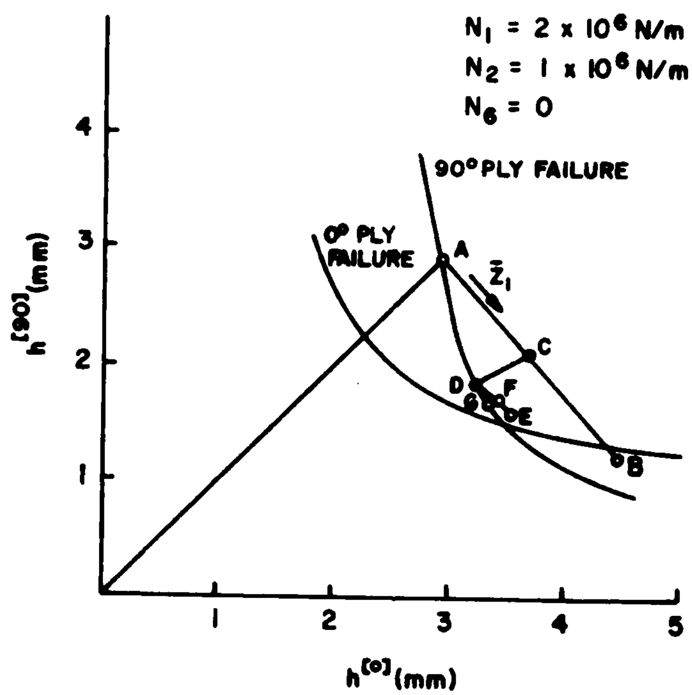


Figure 2. Constraint Surfaces and Optimization Trajectory for 0/90 Under Biaxial Load

The program starts by finding an initial feasible point (A) which lies on a constraint curve farthest from the origin on the line $h^{[90]} = h^{[0]}$. The distance from the origin is calculated using a strain ratio method. Along any vector which passes through the origin, new ply group thickness coordinates are scaled by

$$h_i = h_i^o \cdot S/S^o$$

where S is a scalar distance, h_i^o are the coordinates of the current point and

$$S^o = \left[\sum_{i=1}^{m} (h_i^o)^2\right]^{1/2} .$$

Along this vector, strain can be found using

$$\epsilon_i = \frac{\epsilon_i^o S^o}{S}$$

where ε_i^o is a component of laminate strain evaluated at S_o. Substituting into the failure criteria we have

$$\frac{G_{jk}^{(\theta_i)} \epsilon_j^o \epsilon_k^o S^{o2}}{S^{o2}} + \frac{G_i^{(\theta)} \epsilon_i^o S^o}{S^{o2}} - 1 = 0$$

To ensure the calculated point lies slightly in the feasible region despite any numerical error, the program sets this function equal to the negative of a small number (e_1) rather than zero. Solving this equation for positive S we have

$$S = \frac{-B + \sqrt{(B^2 - 4AC)}}{2A}$$

where

$$A = 1 - e_1$$

$$B = \sum_{j=1}^{3} - G_j^{(\theta_i)} \epsilon_j^o S^o$$

$$C = \sum_{j=1}^{3} \sum_{k=1}^{3} - G_{jk}^{(\theta_i)} \epsilon_j \epsilon_k S^{o2}$$

If S^o lies in the feasible region we solve the above equation for each layer and each load combination, and then take the smallest resulting S as the one that defines the boundary of the feasible region.

The next step in the optimization procedure is to establish a direction vector which will point away from the constraint, point A lies on and is parallel to the plane defined by Σh_i = constant. In Figure 2, this direction is shown as $\overline{Z}$. Finding $\overline{Z}$ first requires calculation of the gradient of the active constraint evaluated at A. Let

$$C_{p,L} = G_{jk}^{(\theta_p)} \epsilon_j^{(L)} \epsilon_k^{(L)} + G_j^{(\theta p)} \epsilon_j^{(L)} - 1$$

where k and N correspond to the layer and load combination of the active constraint. A constraint is considered active if

$$C_{p,L} \geq - e_2$$

where e_2 is a small number. Note, that more than one constraint may be active. The gradient is then given by

$$\vec{\nabla} C_{p,L} = \sum_{i=1}^{m} [G_{jk}^{(\theta p)} (\frac{\partial \epsilon_j^{(L)}}{\partial h_i} \epsilon_k^{(L)} + \epsilon_j^{(L)} \frac{\partial \epsilon_k^{(L)}}{\partial h_i}) + G^{(\theta p)} \frac{\partial \epsilon_i^{(L)}}{\partial h_i}] \hat{h}_i$$

where $\hat{h}_i$ is a unit vector. To find the partials of strain, we start with the basic equation

$$\vec{N} = |A| \vec{\epsilon}$$

$$0 = \frac{\partial}{\partial h_i} |A| \vec{\epsilon} + A \frac{\partial}{\partial h_i} \vec{\epsilon}$$

$$\frac{\partial \vec{\epsilon}}{\partial h_i} = -|A^{-1}| \frac{\partial}{\partial h_i} |A| \vec{\epsilon}$$

and

$$\frac{\partial}{\partial h_i} |A| = \begin{bmatrix} Q_{11}^{(\theta i)} & Q_{12}^{(\theta i)} & Q_{13}^{(\theta i)} \\ Q_{21}^{(\theta i)} & Q_{22}^{(\theta i)} & Q_{23}^{(\theta i)} \\ Q_{13}^{(\theta i)} & Q_{23}^{(\theta i)} & Q_{33}^{(\theta i)} \end{bmatrix} = [Q_i^{(\theta i)}]$$

The gradient vector is normalized to unit length. If more than one constraint is active, the normalized gradients are summed together and the sum is then normalized to one. The negative of the gradient will point away from the constraint, into the feasible region. This vector is now projected onto the plane defined by the unit normal $\hat{n}$, where

$$\hat{n} = \frac{1}{\sqrt{L}} \sum_{i=1}^{m} \hat{h}_i$$

The projection can be made with a double cross product

$$\vec{Z} = \hat{n} \times (-\vec{\nabla} C \times \hat{n})$$

With a vector identify, this can be rewritten as

$$\vec{Z} = (\nabla C \cdot \hat{n})\hat{n} - \vec{\nabla} C$$

Finally, $\vec{Z}$ is also normalized to unit length.

Along $\vec{Z}$, another constraint will eventually be reached (point B in Figure 2). The point is found iteratively by a bisection technique. Since the bisection method is very time consuming, the constraint line is only found within a relatively large error band. What we are really interested in is a point approximately midway between A and B, which is C in the figure. From point C, the strain ratio technique is used to

analytically calculate D. Starting at D, the entire procedure repeats. The program terminates when the distance $\overline{AB}$ or $\overline{CD}$ is small (say 1/10 a ply thickness) or the magnitude of $\vec{Z}$ before normalization is very small (implying $\hat{n}$ and $\vec{\nabla c}$ are almost parallel).

In some cases, $h_k \geq 0$ constraint may be reached. When this happens, that orientation is completely dropped from further calculations. Thus, the constraints associated with a zero thickness layer cannot effect the results. Once an orientation reaches zero thickness, it is never reinstated in later iterations.

Figure 1 shows a case where the program reaches the intersection of two constraints. However, simultaneous failure should not be considered a criteria for optimization. Figure 3 shows a case where only one layer approaches failure. The constraint line for the +45^o layer is completely in the infeasible region. The line $h^{[45]} + h^{[-45]}$ = constant has been included to show that point D is the minimum thickness.

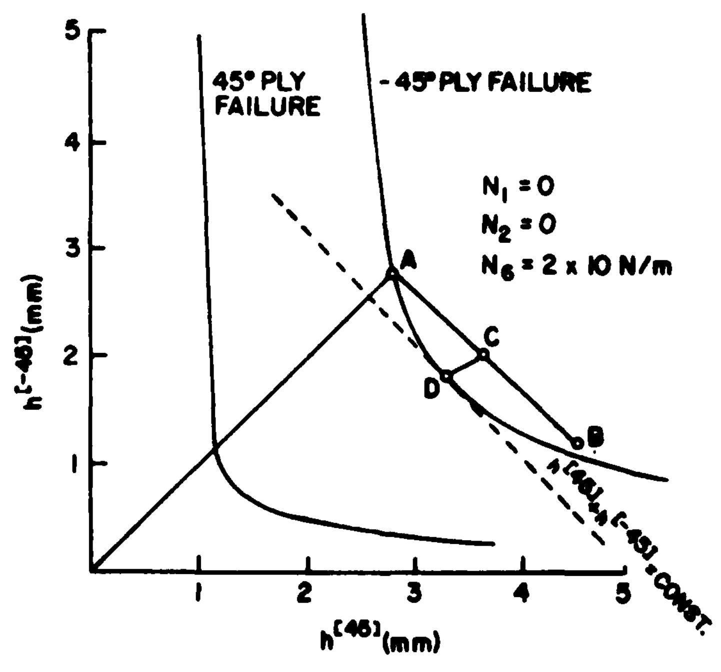


Figure 3. Constraint Surfaces and Optimization Trajectory for $\pm$ 45 Laminate Under Pure Shear

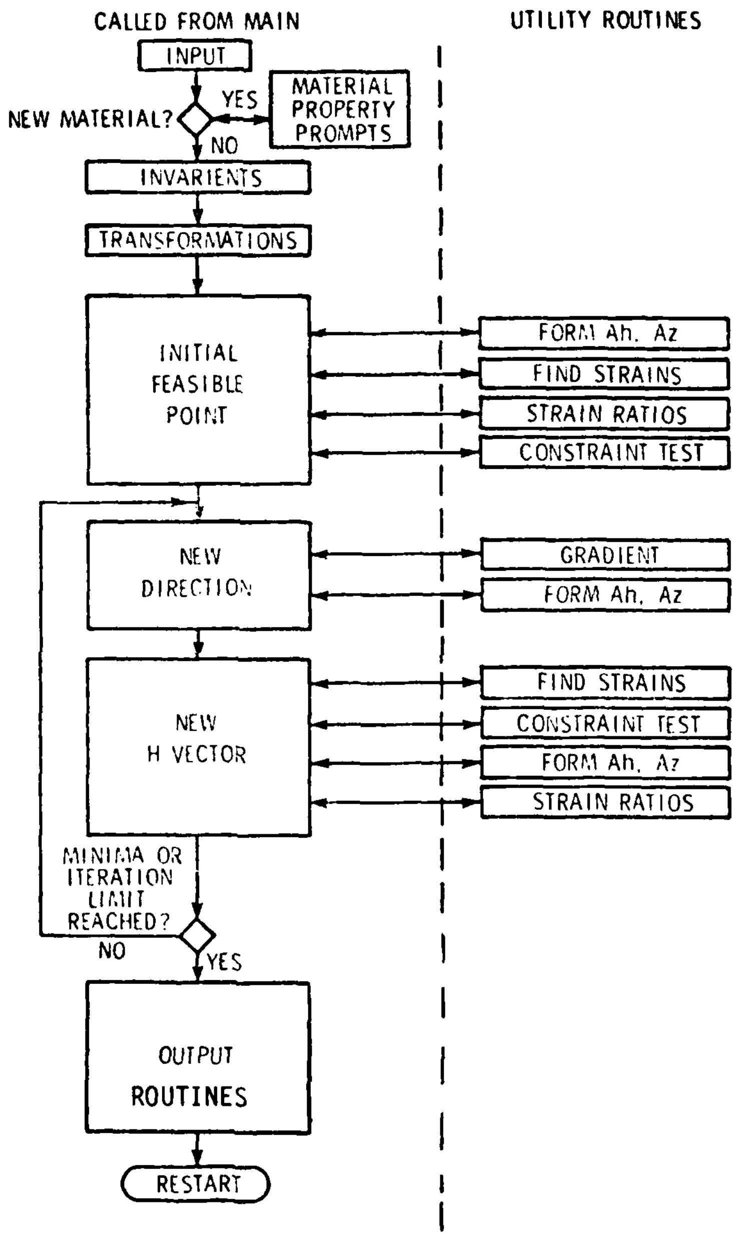
REFERENCES

1. S. W. Tsai and H. T. Hahn, Introduction to Composite Materials, Technomic Publishing Company, Westport, Connecticut, 1980.

2. D. M. Himmelblan, Applied Nonlinear Programming, McGraw-Hill, New York, 1972.

APPENDIX

FLOW CHART



```
10 '** MAIN CLASS**
20 CLEAR 75,330
30 WIDTH 21,5
40 DEFFIL 55,0
50 DEFINT I-P:DEFDBL F
60 DIM A(3,3),B(6,9),C(6
,6),D(3,3),G(3,3),XN(4,3
),AI(3,3),Q(3,3),H(6),R(
3),S(3),T(6),U(5),V(7),X
(6),Y(3),Z(6),E(4,3)
70 DIM C%(10,2)
80 DEF FNDEG(X)=X*57.295
78
90 DEF FNRAD(X)=X/57.295
78
105 RESTORE
110 READ IMAX,E2,E5,E6
120 ITER=1
130 GOSUB 2540
140 PRINT"CORRECTIONS":I
NPUT "(Y/N)";A$
150 IF A$="Y" THEN 130
155 CLS: PRINT"WORKING":
PRINT"ITERATION";ITER
170 GOSUB 2990
180 GOSUB 2330
190 GOSUB 2190
200 GOSUB 1680
205 CLS: PRINT"WORKING":
PRINT"ITERATION";ITER
210 IF F$="FAIL" THEN 33
00
220 GOSUB 1370
230 ITER=ITER+1
240 IF F$="FAIL" OR ITER
>IMAX THEN 3300
250 GOTO 200
```

Comments

20-40 commands to configure the machine

50 Implicit integer and double precision

80-90 convert radians to degrees and degrees to radians

130 - Gosub input

170 - Gosub invariants

180 - Gosub transformations

190 - Gosub initial feasible pt.

200 - Gosub direction

220 - Gosub new thickness

```
260 '** CONSTRAINT TEST*
*
270 G$="PASS": NC=0
280 FOR P=1 TO NPLY
290 IF H(P)=0 THEN 445
300 II=P :GOSUB 1230
310 FOR N=1 TO NL
320 FCON=-1
330 FOR K=1 TO 3
340 FOR J=1 TO 3
350 FCON=FCON+G(K,J)*E(N
,J)*E(N,K)
360 NEXT J
370 FCON=FCON+S(K)*E(N,K
)
380 NEXT K
390 IF FCON>0 THEN G$="F
AIL": RETURN
400 IF FCON<-E5 THEN 440
410 NC=NC+1
420 C%(NC,1)=P
430 C%(NC,2)=N
440 NEXT N
445 NEXT P
450 RETURN
```

290 - if ply thickness zero, ignore constraint

300 - Get G matrix for ply being tested

320 - 380 Solve FCON = $G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i - 1$

410 - 430 If FCON is close to zero identify constraint as active, make list in C% and increment constraint counter

```
460 '** GRADIENT **
475 UNORM=0
480 II=P: GOSUB 1230
490 FOR L=1 TO NPLY
500 IF H(L)=0 THEN 700
510 II=L: GOSUB 1120
520 FOR J=1 TO 3
530 R(J)=0
540 FOR K=1 TO 3
550 R(J)=R(J)-Q(J,K)*E(N
,K)
560 NEXT K,J
570 FOR J=1 TO 3
580 Y(J)=0
590 FOR K=1 TO 3
600 Y(J)=Y(J)+AI(J,K)*R(
K)
610 NEXT K,J
620 Z(L)=0
630 FOR J=1 TO 3
640 FOR K=1 TO 3
650 Z(L)=Z(L)+G(J,K)*(Y(
J)*E(N,K)+E(N,J)*Y(K))
660 NEXT K
670 Z(L)=Z(L)+S(J)*Y(J)
680 NEXT J
690 UNORM=UNORM+Z(L)*Z(L
)
700 NEXT L
710 UNORM=SQR(UNORM)
720 FOR L=1 TO NPLY
730 Z(L)=Z(L)/UNORM
740 NEXT L
760 RETURN
770 '** STRAINS **
780 DIM F(3,3)
790 FOR I=1 TO 3
800 FOR J=1 TO 3
810 F(I,J)=A(I,J)+D(I,J)
*S
820 NEXT J,I
830 DET#=F(1,1)*F(2,2)*F
(3,3)+2*F(1,2)*F(2,3)*F(
1,3)-F(2,2)*F(1,3)*F(1,3
)-F(1,1)*F(2,3)*F(2,3)-F
(3,3)*F(1,2)*F(1,2)
840 AI(1,1)=(F(2,2)*F(3,
3)-F(2,3)*F(2,3))/DET#
850 AI(2,2)=(F(1,1)*F(3,
3)-F(1,3)*F(1,3))/DET#
860 AI(1,2)=(F(1,3)*F(2,
3)-F(1,2)*F(3,3))/DET#
870 AI(3,3)=(F(1,1)*F(2,
2)-F(1,2)*F(1,2))/DET#
880 AI(1,3)=(F(1,2)*F(2,
3)-F(2,2)*F(1,3))/DET#
890 AI(2,3)=(F(1,2)*F(1,
3)-F(1,1)*F(2,3))/DET#
900 AI(2,1)=AI(1,2):AI(3
,2)=AI(2,3):AI(3,1)=AI(1
,3)
910 ERASE F
920 FOR I=1 TO NL
930 FOR J=1 TO 3
940 E(I,J)=0
950 FOR K=1 TO 3
960 E(I,J)=E(I,J)+AI(J,K
)*XN(I,K)
970 NEXT K,J,I
980 RETURN
```

480 - Get G matrix for designated ply

510 - For each ply, get Q matrix

540 - 560 $\vec{R} = - \frac{\partial}{\partial h} A \vec{\varepsilon}$

580 - 610 $\vec{Y} = |A^{-1}| \vec{R}$

$$Y = \frac{\partial}{\partial h_k} \vec{\varepsilon}$$

620 - 680 $\vec{\nabla}$ (FCON) $= [G_{ij}(\varepsilon_i \frac{\partial \varepsilon j}{\partial h_k} + \frac{\partial \varepsilon i}{\partial h_k} \varepsilon_j) + G_i (\frac{\partial \varepsilon i}{\partial h_k})]\ \hat{h}k$

690 - 730 Normalize $\vec{\nabla}$(FCON)

790 - 820 "F" is the A matrix corresponding to a point S along the Z vector

830 - 900 invert A

920 - 970 Solve $\vec{\varepsilon} = |A^{-1}|\ \vec{N}$ for each independent loading

```
990 '** A MATRIX **
1000 FOR I= 1 TO 3
1010 FOR J=1 TO 3
1020 A(I,J)=0: D(I,J)=0
1030 NEXT J,I
1040 FOR I=1 TO NPLY
1050 II=I: GOSUB 1120
1060 FOR J=1 TO 3
1070 FOR K=1 TO 3
1080 A(J,K)=A(J,K)+Q(J,K
)*H(I)
1090 D(J,K)=D(J,K)+Q(J,K
)*Z(I)
1100 NEXT K,J,I
1110 RETURN
```

1000 - 1100 The matrix D is formed so that along the Z vector

$$|A| = |\tilde{A}| + |D| \cdot S$$

where S is a scalar

```
1120 '** FORM Q **
1130 Q(1,1)=C(II,1)
1140 Q(1,2)=C(II,3)
1150 Q(1,3)=C(II,5)
1170 Q(3,1)=C(II,5)
1180 Q(3,2)=C(II,6)
1190 Q(3,3)=C(II,4)
1195 Q(2,3)=C(II,6)
1200 Q(2,2)=C(II,2)
1210 Q(2,1)=C(II,3)
1220 RETURN
```

1130 - 1210 Convert C array into 3 x 3 Q matrix for ply designated by II

```
1230 '** FORM G **
1240 G(1,1)=B(II,1)
1250 G(1,2)=B(II,3)
1260 G(1,3)=B(II,5)
1270 G(2,1)=B(II,3)
1280 G(2,2)=B(II,2)
1290 G(2,3)=B(II,6)
1300 G(3,1)=B(II,5)
1310 G(3,2)=B(II,6)
1320 G(3,3)=B(II,4)
1330 S(1)=B(II,7)
1340 S(2)=B(II,8)
1350 S(3)=B(II,9)
1360 RETURN
```

1240 - 1350 Convert B array into 3 x 3 G matrix for ply designated by II. Linear failure terms placed in vector S

```
1370 '** NEW H VECTOR **
1380 SMAX=1E10
1390 FOR I=1 TO NPLY
1400 IF Z(I)<>0 THEN S=-
H(I)/Z(I)
1410 IF S>0 AND S<SMAX T
HEN SMAX=S
1420 NEXT I
1430 F$=""
1440 IF SMAX> 10 THEN F$
="FAIL": RETURN
1450 S1=0: S2=SMAX: S=SM
AX
1460 IF NC=0 THEN 1590
1470 GOSUB 770: GOSUB 26
0
1480 IF G$="FAIL" THEN S
2=S ELSE S1=S
1490 IF S1=SMAX THEN 153
5
1500 S=(S1+S2)/2
1510 IF S2-S1<E2 AND S1=
0 THEN F$="FAIL": S=0: G
OTO 1650
1520 IF S1/(S2-S1)<4 THE
N 1470
```

1380 - 1420 Find distance along $\vec{Z}$ to find $h_i = 0$ constraint

1450 - 1500 Bisection method to find distance to next constraint. If no constraints violated at S = SMAX then stop search

```
1530 S=S/2
1535 SREF=0
1540 FOR I=1 TO NPLY
1550 H(I)=H(I)+Z(I)*S
1560 IF H(I)<E2 THEN H(I
)=0
1570 SREF=SREF+H(I)*H(I)
1580 NEXT I
1590 S=0:SREF=SQR(SREF)
1600 GOSUB 990: GOSUB 77
0: GOSUB 2020
1610 IF SREF-S<E2 THEN F
$="FAIL"
1620 FOR I=1 TO NPLY
1630 H(I)=H(I)*S/SREF
1640 NEXT I
1650 S=0
1660 GOSUB 990: GOSUB 77
0: GOSUB 260
1670 RETURN
1680 '** DIRECTION **
1690 Z=0: UNORM=1
1700 FOR I=1 TO NPLY
1710 X(I)=0
1720 Z=Z+SGN(H(I))
1730 NEXT I
1740 Z=1/SQR(Z)
1750 IF NC=0 THEN 1860
1760 FOR I=1 TO NC
1770 P=C%(I,1): N=C%(I,2
)
1780 GOSUB 460
1790 FOR J=1 TO NPLY
1800 LET X(J)=X(J)-Z(J)
1810 NEXT J,I
1815 UNORM=0
1820 FOR J=1 TO NPLY
1830 UNORM=UNORM+X(J)*X(
J)
1840 NEXT J
1850 UNORM=SQR(UNORM): T
EST=0
1860 FOR I=1 TO NPLY
1870 X(I)=X(I)/UNORM
1880 TEST=TEST+X(I)*Z*SG
N(H(I))
1890 NEXT I
1900 UNORM=0
1910 FOR I=1 TO NPLY
1920 Z(I)=X(I)-TEST*Z*SG
N(H(I))
1930 UNORM=UNORM+Z(I)*Z(
I)
1940 NEXT I
1950 IF UNORM<1E-6 THEN
F$="FAIL": RETURN  ELSE
F$=""
1960 UNORM=SQR(UNORM)
1970 FOR I=1 TO NPLY
1980 Z(I)=Z(I)/UNORM
1990 NEXT I
2000 GOSUB 990
2010 RETURN
```

1530 - 1600 at point halfway between constraints, use strain ratio routine to find how much the laminate thickness can be reduced

1610 If change in thickness small, set flag to halt program

1620 - 1660 Update h vector, A matrix, strains

1760 - 1840 For each active constraint call gradient subroutine. Sum negative of each gradient into $\vec{X}$ and normalize $\vec{X}$

1860 - 1890 Take dot product of $\vec{X}$ and unit normal to Σhi = const. plane

1910 - 1940 $\vec{Z}$ is a vector parallel to the Σhi = const. plane and pointing away from the active constraints

1950 if the magnitude of $\vec{Z}$ is very small, a local minima has been reached

```
2020 '** STRAIN RATIO **
2030 FOR P=1 TO NPLY
2040 IF H(P)=0 THEN 2160
2050 II=P: GOSUB 1230
2060 FOR N=1 TO NL
2070 B#=0:C#=0
2080 FOR I=1 TO 3
2090 FOR J=1 TO 3
2100 C#=C#-SREF*SREF*G(I
,J)*E(N,I)*E(N,J)
2110 NEXT J
2120 B#=B#-SREF*S(I)*E(N
,I)
2130 NEXT I
2140 SVAL=(-B#+SQR (B#*B
#-4*C#*(1-E6)))/(2*(1-E6
))
2150 IF SVAL>S THEN S=SV
AL
2155 NEXT N
2160 NEXT P
2180 RETURN
2190 '** IFP **
2200 Z=1/SQR(NPLY)
2210 FOR I=1 TO NPLY
2220 Z(I)=Z: H(I)=Z
2230 NEXT I
2240 GOSUB 990
2250 S=0: SREF=1
2260 GOSUB 770: GOSUB 20
20
2270 FOR I=1 TO NPLY
2280 H(I)=H(I)*S
2290 NEXT I
2300 S=0
2310 GOSUB 990: GOSUB 77
0: GOSUB 260
2320 RETURN
2330 '** TRANSFORM **
2340 FOR I=1 TO NPLY
2350 C2=COS(2*T(I)): C4=
COS(4*T(I))
2360 S2=SIN(2*T(I)): S4=
SIN(4*T(I))
2370 B(I,1)=U(1)+C2*U(2)
+C4*U(3)
2380 B(I,2)=U(1)-C2*U(2)
+C4*U(3)
2390 B(I,3)=U(4)-C4*U(3)
2400 B(I,4)=U(5)-C4*U(3)
2410 B(I,5)=S2/2*U(2)+S4
*U(3)
2420 B(I,6)=S2/2*U(2)-S4
*U(3)
2430 B(I,7)=U(6)+C2*U(7)
2440 B(I,8)=U(6)-C2*U(7)
2450 B(I,9)=S2*U(7)
2460 C(I,1)=U(1)+C2*U(2)
+C4*U(3)
2470 C(I,2)=U(1)-C2*U(2)
+C4*U(3)
2480 C(I,3)=U(4)-C4*U(3)
2490 C(I,4)=U(5)-C4*U(3)
2500 C(I,5)=S2/2*U(2)+S4
*U(3)
2510 C(I,6)=S2/2*U(2)-S4
*U(3)
2520 NEXT I
2530 RETURN
```

2030 - 2140 For each possible constraint solve for S in

$$G_{ij}\varepsilon_i\varepsilon_j \frac{(SREF)^2}{S^2} + G_i\varepsilon_i \frac{(SREF)}{S} - 1 = -E6$$

2150 Take smallest value (corresponds to closest constraint)

2200 - 2310 For equal ply ratios, find the smallest laminate thickness which does not violate any constraints. Initialize A matrix, strains, and constraint list

2370 - 2450 Transform failure parameters in following order

$B(I,1)=G_{11}$	$B(I,5)=G_{16}$
$B(I,2)=G_{22}$	$B(I,6)=G_{26}$
$B(I,3)=G_{12}$	$B(I,7)=G_{1}$
$B(I,4)=G_{66}$	$B(I,8)=G_{2}$
	$B(I,9)=G_{3}$

2460 - 2510 Transform modulus in following order

$C(I,1) = Q_{11}$	$C(I,5) = Q_{16}$
$C(I,L)=Q_{22}$	$C(I,6)=Q_{26}$
$C(I,3)=Q_{12}$	
$C(I,4)=Q_{66}$	

```
2540 '*** INPUT ***
2550 CLS
2600 PRINT "PRESS ANY KE
Y WHEN":PRINT"DESIRED MA
TERIAL":PRINT"APPEARS"
2610 FOR K=1 TO 750: NEX
T
2620 FOR M=0 TO 6
2640 IF M=6 THEN M$="NEW
 MATERIAL" ELSE GET%M,EX
,EY,UX,ES,TPLY,XT,YT,XC,
YC,SS,M$
2650 CLS:PRINT M$:SOUND2
0,1
2660 FOR J=1 TO 200
2670 IF INKEY$<>"" THEN
2700
2675 NEXT J,M
2680 GOTO 2620
2700 IF M=6 THEN GOSUB 9
000:GOTO 2600
2705 CLS:PRINT "* ";M$;"
 *"
2710 PRINT "HOW MANY"
2720 INPUT "PLY GROUPS";
NPLY
2730 CLS: PRINT"ENTER PL
Y GROUP"
2740 PRINT"ORIENTATIONS"
2750 FOR I=1 TO 200
2760 NEXT I
2770 CLS
2780 FOR I=1 TO NPLY
2790 PRINT "PLY ";I
2800 INPUT T(I)
2810 T(I)=FNRAD(T(I))
2820 NEXT I
2830 PRINT"ENTER NUMBER
OF"
2840 PRINT "INDEPENDENT
LOAD"
2850 INPUT "CONDITIONS";
NL
2900 FOR I=1 TO NL
2910 CLS: PRINT "Load ";
I;" in MPa."
2920 INPUT "N1=";XN(I,1)
2930 INPUT "N2=";XN(I,2)
2940 INPUT "N6=";XN(I,3)
2950 FOR J=1 TO 3
2960 XN(I,J)=XN(I,J)*1E6
2970 NEXT J,I
2980 RETURN
```

2600 - 2675 List available materials. Get% is an HX-20 command to get data from a non-volatile RAM file

```
2990 '** INVARIENTS **
3050 VY=1/(1-UX*UX*EY/EX
)
3060 QXX=UY*EX*1E9: QYY=
UY*EY*1E9
3070 QXY=UY*UX*EY*1E9: Q
S=ES*1E9
3080 U(1)=(3*QXX+3*QYY+2
*QXY+4*QS)/8
3090 U(2)=(QXX-QYY)/2
3100 U(3)=(QXX+QYY-2*QXY
-4*QS)/8
3110 U(4)=(QXX+QYY+6*QXY
-4*QS)/8
3120 U(5)=(QXX+QYY-2*QXY
+4*QS)/8
3130 EX=1E-12/(XT*XC): E
Y=1E-12/(YT*YC): ES=1E-1
2/(SS*SS)
3140 FX=(1/XT-1/XC)/1E6:
 FY=(1/YT-1/YC)/1E6
3150 EXY=-SQR (EX*EY)/2
3160 GXX=EX*QXX*QXX+2*EX
Y*QXX*QXY+EY*QXY*QXY
3170 GYY=EX*QXY*QXY+2*EX
Y*QXY*QYY+EY*QYY*QYY
3180 GXY=EX*QXX*QXY+EXY*
(QXX*QYY+QXY*QXY)+EY*QXY
*QYY
3190 GSS=ES*QS*QS
3200 GX=FX*QXX+FY*QXY
3210 GY=FX*QXY+FY*QYY
3220 U(1)=(3*GXX+3*GYY+2
*GXY+4*GSS)/8
3230 U(2)=(GXX-GYY)/2
3240 U(3)=(GXX+GYY-2*GXY
-4*GSS)/8
3250 U(4)=(GXX+GYY+6*GXY
-4*GSS)/8
3260 U(5)=(GXX+GYY-2*GXY
+4*GSS)/8
3270 U(6)=(GX+GY)/2
3280 U(7)=(GX-GY)/2
3290 RETURN
```

3050 - 3280 Calculate invariants for use in transformations. Note that some variables like EX and EY get reused, so their value may not be what you might expect after routine is called

```
3300 '** OUTPUT **
3302 SOUND 15,2:SOUND50,
2
3305 K$="Hit any key to
cont.":U$="MN/m"
3310 CLS: TEST=0
3320 FOR I=1 TO NPLY
3330 TEST=TEST+H(I): NEX
T I
3350 PRINT "TOTAL THICKN
ESS="
3360 PRINT TEST;" m."
3370 PRINT USING "####.#
# Plies";TEST/TPLY
3375 PRINT K$;
3380 A$=INKEY$:IF A$<>""
 THEN 3380
3390 IF INKEY$="" THEN 3
390
3400 CLS:PRINT"Press Y i
f printout","of displaye
d result","is desired. P
ress N","if not";
3410 FOR I= 1 TO 800:NEX
T I
3415 A$=INKEY$:IF A$<>""
 THEN 3415
3420 CLS: RESTORE 6120
3425 J=0:A$=INKEY$
3430 FOR I=1 TO 8
3440 READ A$:CLS:PRINT:P
RINT A$:SOUND20,1
3445 A$=INKEY$:IFA$="" T
HEN 3445
3450 PRINT A$;:FOR KK=1
TO 75:NEXT KK
3455 IF A$="Y" THEN J=J+
1:C%(J,1)=I
3460 NEXT I
3464 IF J<>0 THEN LPRINT
 STRING$(24,"▒")
3465 FOR K=1 TO J
3470 ON C%(K,1) GOSUB 50
00,5200,4000,4200,4400,4
600,4800,8000
3485 LPRINT
3490 NEXT K
3495 CLS:PRINT"FINISHED"
,K$
3496 IF INKEY$=""THEN349
6 ELSE RUN
```

```
4000 '** PLY RATIO**
4002 CLS:LPRINT "Total t
hickness="
4004 LPRINT USING ".####
^^^^ m. ";TEST
4006 LPRINT USING "####.
## Plies";TEST/TPLY
4008 LPRINT
4030 A$="ANGLE  RATIO  #
PLIES"
4040 LOCATE 0,1:PRINT A$
:LPRINT A$
4050 FOR I=1 TO NPLY
4060 A=CINT((FNDEG(T(I))
*1E2))/1E2
4070 B=CINT((H(I)/TEST*1
E4))/1E4
4080 C=CINT((H(I)/TPLY*1
E2))/1E2
4090 PRINT A;TAB(6);B;TA
B(13);C
4100 LPRINT A;TAB(6);B;T
AB(13);C
4120 NEXT I
4150 RETURN
4200 '** STRENGTH**
4210 LPRINT "STRENGTH RA
TIOS"
4215 LPRINT "1=ULTIMATE
STRAIN":
4220 LPRINT ">1 IS SAFE"
4225 FOR I=1 TO NL
4230 LPRINT "LOADING "I
4235 LPRINT "PLY"
4240 FOR P=1 TO NPLY
4245 IF H(P)=0 THEN 4305
4250 II=P:GOSUB 1230
4255 A#=0:B#=0
4260 FOR J=1 TO 3
4265 FOR K=1 TO 3
4270 A#=A#+G(J,K)*E(I,J)
*E(I,K)
4275 NEXT K
4280 B#=B#+S(J)*E(I,J)
4285 NEXT J
4290 A#=(-B#+SQR(B#*B#+4
*A#))/(2*A#)
4295 A=FIX(A#*1E4+.5)/1E
4
4300 LPRINT FNDEG(T(P));
TAB(10);A
4305 NEXT P,I
4310 RETURN
4400 '**STRAINS**
4410 LPRINT TAB(4);"LAMI
NATE STRAINS"
4420 FOR N=1 TO NL
4430 LPRINT "LOADING "N
4440 LPRINT USING "e1=+#
.###E-03";E(N,1)*1E3
4450 LPRINT USING "e2=+#
.###E-03";E(N,2)*1E3
4460 LPRINT USING "e6=+#
.###E-03";E(N,3)*1E3
4465 NEXT N
4470 RETURN
4600 ' **A MATRIX**
4610 CLS
4620 LPRINT"Norm. |A| in
 GPa."
4630 FOR I=1 TO 3
4640 FOR J=1 TO 3
4650 D(I,J)=A(I,J)/1E9/T
EST
4660 NEXT J,I
4670 GOSUB 7000
4680 RETURN
```

4210 - 4305 Strength ratio is defined as the value of R in

$$G_{ij}\varepsilon_i\varepsilon_j R^2 + G_i\varepsilon_i R - 1 = 0$$

```
4800 'A INVERSE
4810 LPRINT"Compliance
(normalized)"
4820 LPRINT"in 1/TPa."
4830 FOR I=1 TO 3
4840 FOR J=1 TO 3
4850 D(I,J)=AI(I,J)*TEST
*1E12
4860 NEXT J,I
4870 GOSUB 7000
4880 RETURN
5000 LPRINT "Material Pr
operties"
5010 GET%M,EX,EY,UX,ES,T
PLY,XT,YT,XC,YC,SS,M$
5015 LPRINT M$
5020 LPRINT "EX=";EX;"GP
a"
5030 LPRINT "EY=";EY;"GP
a"
5040 LPRINT "ES=";ES;"GP
a"
5050 LPRINT "UX=";UX
5060 LPRINT "X=";XT;"MPa
"
5070 LPRINT "X'=";XC;"MP
a"
5072 LPRINT "Y=";YT;"MPa
"
5074 LPRINT "Y'=";YC;"MP
a"
5080 LPRINT "S=";SS;"MPa
"
5090 LPRINT "Ply Thickne
ss";TPLY"m"
5095 RETURN
6000 DATA 10,5E-5,.1,1E-
6
6120 DATA Ply properties
,Loads,Total thickness &
    ply ratios,Strength
ratios
6130 DATA Laminate strai
ns,Stiffness matrix,Comp
liance matrix,Ply ratio
graph
7000 'FANCY
7010 LPRINT "┌────────┬───
──────┬────────┐"
7020 LPRINT USING "|###.
###";D(1,1),D(1,2),D(1,3
)
7030 A$="├────────┼──────
─┼────────┤"
7040 LPRINT A$
7050 LPRINT USING "|###.
###";D(2,1),D(2,2),D(2,3
)
7060 LPRINT A$
7070 LPRINT USING "|###.
###";D(3,1),D(3,2),D(3,3
)
7080 LPRINT "└────────┴───
──────┴────────┘"
7100 RETURN
```

```
8000 LPRINT "**Ply Ratio
 Graph**"
8010 LPRINT "ANGLE"
8020 Z=0:CLS
8030 FOR I=1 TO NPLY
8040 X(I)=H(I)/TEST
8050 IF X(I)>Z THEN Z=X(
I)
8055 NEXT
8060 DELTA=CINT(Z/.08)/1
00
8070 A=CINT(DELTA*1000)
8080 IF A<>20AND A<>50AN
D A<>100AND A<>150 THEN
DELTA=DELTA+.01:GOTO 807
0
8090 FOR I=1 TO NPLY
8100 Z=(X(I)/DELTA*12)+2
5
8110 II=CINT(ABS(COS(I/2
*3.14159)))
8115 IF Z<26 THEN 8160
8130 FOR K=8 TO 15
8140 A=K+II*16:LINE(26,A
)-(Z,A),PSET
8150 NEXT K
8160 LOCATE 0,1+II*2:PRI
NT USING "+###";FNDEG(T(
I)):LOCATE0,0
8170 IF II=1 THEN COPY:C
LS
8180 NEXT
8190 LPRINT TAB(4);"| |
| | | | | | |":LPRINT TA
B(6);"↑":LPRINTUSING"
  .##";DELTA:LPRINT TAB(
9);"PLY RATIO"
8200 RETURN
9000 PRINT"REVIEW OR NEW
"
9010 INPUT "DATA (R/N)";
A$
9020 IF A$="R" THEN 9190
9030 PRINT"WHICH MATERIA
L":PRINT"WILL YOU"
9040 INPUT "REPLACE (0-5
)";I
9050 INPUT "EX(GPa)=";EX
9060 INPUT "EY(GPa)=";EY
9070 INPUT "UX=";UX
9075 INPUT "ES(GPa)=";ES
9080 INPUT "X(MPa)=";X
9090 INPUT "X'(MPa)=";XX
9100 INPUT "Y(MPa)=";Y
9110 INPUT "Y'(MPa)=";YY
9120 INPUT "S(MPa)=";S
9130 INPUT "THICKNESS (m
)=";TPLY
9140 INPUT "NAME (15 CHR
. MAX.)";M$
9150 PUT%I,EX,EY,UX,ES,T
PLY,X,Y,XX,YY,S,M$
9160 PRINT "ADDITIONAL":
INPUT "CHANGES (Y/N)";A$
9170 IF A$="Y" THEN 9000
9180 RETURN
9190 PRINT"REVIEW WHICH"
:INPUT"MATERIAL (0-5)";M
9200 GOSUB5000
9210 GOTO 9160
```

8020 - 8080 Automatic scaling

9150 - Put% is an HX-20 command to place a data string into a non-volatile RAM file

```
9500 OPEN "I",#1,"CAS1:D
ATA"
9510 FOR I=0 TO 5
9520 INPUT #1,EX,EY,VX,E
S,T,X,Y,XX,YY,S,M$
9530 PUT%I,EX,EY,VX,ES,T
,X,Y,XX,YY,S,M$
9540 NEXT
9550 CLOSE #1
9560 DELETE 45
```

9500 - 9550 Routine to read RAM file data from tape called immediately upon loading program and has no further function

9560 - Line 45 reads GOTO 9500 and is not needed after data is read

```
5 DEFFIL 55,0
10 '**SAVE RAM FILE
20 OPEN "O",#1,"CAS1:DAT
A"
30 FOR I=0 TO 5
40 GET%I,EX,EY,VX,ES,T,X
,Y,XX,YY,S,M$
50 PRINT #1,EX;EY;VX;ES;
T;X;Y;XX;YY;S;M$
60 NEXT
70 CLOSE #1
```

9570 - Routine used to load RAM file onto a tape after program has been stored. Usually placed in a separate program area from main program (using command LOGIN 2). Only needed if additional tape copies are being made.